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Publisher: Taylor & Francis

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## Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/gmcl16>

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Version of record first published: 14 Oct 2011.

To cite this article: S. Alexander, J. Bernasconi, W. R. Schneider, R. Biller & R. Orbach (1982): Frequency-Dependent Conductivity of Quasi-One-Dimensional Electronic Conductors, *Molecular Crystals and Liquid Crystals*, 85:1, 121-128

To link to this article: <http://dx.doi.org/10.1080/00268948208073637>

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*Mol. Cryst. Liq. Cryst.*, 1982, Vol. 85, pp. 121-128  
0026-8941/82/8501-0121\$06.50/0  
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Printed in the United States of America

(Proceedings of the International Conference on Low-Dimensional Conductors, Boulder, Colorado, August 1981)

## FREQUENCY-DEPENDENT CONDUCTIVITY OF QUASI-ONE-DIMENSIONAL ELECTRONIC CONDUCTORS

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Received for publication August 31, 1981

For a one-dimensional model system, described by a master equation with random hopping rates  $W_{n,n+1} \propto \exp(-\Delta_{n,n+1}/T)$ , the frequency and temperature dependent electrical conductivity  $\sigma(\omega, T)$  is calculated within the framework of an Effective-Medium approximation. Using a specific distribution of barrier heights,  $\hat{p}(\Delta) \propto \exp(-\Delta/T_m)$  for  $\Delta_{\min} < \Delta < \Delta_{\max}$ , the model is shown to account quantitatively for the measured (Zettl, Grüner, and Clark) frequency and temperature dependence of  $\text{Re}\sigma$  and  $\text{Im}\sigma$  in the quasi-one-dimensional conductor  $\text{Qn}(\text{TCNQ})_2$ .

## I. INTRODUCTION

Peculiar frequency and temperature dependences of the complex electrical conductivity,  $\sigma(\omega, T)$ , are observed almost universally in quasi-one-dimensional conductors<sup>1-4</sup>. The dc conductivity,  $\text{Re } \sigma(0, T)$ , of such systems is generally activated, and  $\text{Re } \sigma(\omega, T)$  appears to increase from its dc value at a temperature dependent crossover frequency

$\omega_{c.p.}(T)$ . It saturates at a constant value,  $\text{Re } \sigma(\infty, T)$ , at high frequencies, and the ratio  $\text{Re } \sigma(\infty, T)/\text{Re } \sigma(0, T)$  decreases towards one with increasing temperature. The dielectric constant,  $\epsilon(\omega, T) \propto -\text{Im } \sigma(\omega, T)/\omega$ , is strongly frequency dependent, and exhibits a pronounced peak as a function of temperature.

We have shown<sup>5</sup> that disorder, combined with one-dimensional transport, can account for all these features, and in the following we shall summarize some details and consequences of our investigations. Our simple one-dimensional model is a variant of the random barrier model<sup>6</sup> introduced originally to describe the anomalous behavior of  $\sigma(\omega, T)$  in the one dimensional superionic conductor hollandite. It is based on the existence of impurity-induced potential barriers which divide the one-dimensional system into segments of average length  $L_0$ , and on the assumption that for all frequencies and temperatures of interest the intrinsic segment conductance is much larger than the intersegment transfer rates

$$W_{n,n+1} = \omega_{at} \exp(-\Delta_{n,n+1}/T) \quad . \quad (1)$$

Here,  $\Delta_{n,n+1}$  represents the barrier height, and  $\omega_{at}$  is an attempt frequency. The transport properties of the system are then completely dominated by the probability distribution  $\rho(w)$  of the (mutually independent) transfer rates  $W_{n,n+1}$ , and can be described<sup>6,7</sup> by a master equation of the form

$$\frac{dP_n}{dt} = W_{n,n-1}(P_{n-1} - P_n) + W_{n,n+1}(P_{n+1} - P_n) \quad , \quad (2)$$

where  $P_n$  denotes the excess charge on segment  $n$ .

The properties of random one-dimensional systems of this type have recently been investigated in some detail<sup>7-9</sup>. The conductivity  $\sigma(\omega, T)$  corresponding to these model systems can be written as<sup>8,9</sup>

$$\sigma(\omega, T) = \frac{n_0 e^2 L_0^2}{kT} \langle D(-i\omega) \rangle \quad , \quad (3)$$

where the generalized diffusion constant  $\langle D(z) \rangle$  is given by

$$\langle D(z) \rangle = \frac{1}{2} z^2 \sum_{n=-\infty}^{\infty} n^2 \langle \tilde{P}_n(z) \rangle \quad (4)$$

Here  $n_0$  is the density and  $e$  the charge of the charge carriers, and  $\langle \dots \rangle$  denotes an average with respect to the probability distribution of the  $W_{n,n+1}$ . The  $\tilde{P}_n(z)$  are determined by

$$z\tilde{P}_n + W_{n,n-1}(\tilde{P}_n - \tilde{P}_{n-1}) + W_{n,n+1}(\tilde{P}_n - \tilde{P}_{n+1}) = \delta_{n,0} \quad (5)$$

i.e. by the Laplace transform of Eq.(2), subject to the initial condition  $P_n(0) = \delta_{n,0}$ .

## II. EFFECTIVE-MEDIUM APPROXIMATIONS

To investigate the behavior of  $\langle D(-i\omega) \rangle$  over the whole frequency range, Effective-Medium-type approximations have been proved to be very useful<sup>6,8,9</sup>. The random  $W_{n,n+1}$  in Eq.(5) are replaced by an effective, frequency dependent transfer rate  $W_{\text{eff}}(z)$ , which is determined by a suitably chosen self-consistency equation. It follows that  $\langle D(z) \rangle$  is approximated by  $W_{\text{eff}}(z)$ , and we have analyzed<sup>8,9</sup> two different self-consistency conditions. One of them is obtained by applying Kirkpatrick's resistor network Effective-Medium theory<sup>10</sup> to the electrical network analog corresponding to Eq.(5)<sup>8,9</sup>. It is equivalent to that obtained by the Single-Bond Coherent Potential Approximation and leads to

$$\int_0^{\infty} dw \rho(w) \frac{w - W_{\text{eff}}}{w + \frac{1}{2}(g_{\text{eff}} + z)} = 0 \quad (6)$$

where  $W_{\text{eff}}$  and  $g_{\text{eff}}$  are related by

$$g_{\text{eff}} = \left[ \frac{1}{W_{\text{eff}}} + \frac{1}{g_{\text{eff}} + z} \right]^{-1} \quad (7)$$

We note that  $z$ ,  $g_{\text{eff}}$ , and  $W_{\text{eff}}$  are complex quantities, and that Eqs.(6) and (7) can easily be solved numerically for  $W_{\text{eff}}(z)$ .

For arbitrary  $\rho(w)$ , this Effective-Medium approximation reproduces the exact  $\omega \rightarrow \infty$  asymptotic behavior<sup>9</sup> of the

real and imaginary part of  $\langle D(-i\omega) \rangle$ . For  $\omega \rightarrow 0$  it leads to the same  $\omega$ -dependence for  $\langle D(-i\omega) \rangle$  as has been obtained<sup>8,9</sup> from a general scaling hypothesis for the  $\langle \tilde{P}_n(z) \rangle$ . We therefore expect that  $W_{\text{eff}}(-i\omega)$  represents a very accurate approximation to  $\langle D(-i\omega) \rangle$  over the entire frequency range.

### III. THE BARRIER HEIGHT DISTRIBUTION

Within our model, the detailed behavior of  $\sigma(\omega, T)$  is completely determined by the properties of the transfer rate distribution  $\rho(w)$  which, through Eq.(1), arises from a barrier height distribution  $\hat{p}(\Delta)$ . For our subsequent discussion, we shall choose the specific form

$$\hat{p}(\Delta) \propto \exp(-\Delta/T_m) \quad , \quad \Delta_{\min} < \Delta < \Delta_{\max} \quad , \quad (8)$$

which leads to

$$\rho(w) \propto w^{-\alpha(T)} \quad , \quad W_{\min} < w < W_{\max} \quad , \quad (9)$$

where  $\alpha = 1 - T/T_m$ ,  $W_{\min} = \omega_{\text{at}} \exp(-\Delta_{\max}/T)$ ,  $W_{\max} = \omega_{\text{at}} \cdot \exp(-\Delta_{\min}/T)$ , and where the probability densities  $\hat{p}(\Delta)$  and  $\rho(w)$  are, of course, properly normalized.

A barrier height distribution of this type with  $\Delta_{\max} \rightarrow \infty$ , has been shown<sup>6</sup> to lead to a remarkably accurate and detailed description of the anomalous properties of  $\sigma(\omega, T)$  in the one-dimensional superionic conductor hollandite. In the following we shall demonstrate that the truncated distribution of Eq.(8), with finite values for  $\Delta_{\min}$  and  $\Delta_{\max}$ , also leads to a reasonably accurate description of both the frequency and temperature dependence of  $\sigma(\omega, T)$  in the quasi-one-dimensional electronic conductor  $\text{Qn(TCNQ)}_2$ . We note, however, that in this case the quality of the description is not very sensitive to the precise analytic form of  $\hat{p}(\Delta)$ , in contrast to the situation in hollandite<sup>6</sup>.

For the specific, truncated probability density  $\rho(w)$  of Eq.(9), our Effective-Medium analysis of section II leads to the following prediction for the asymptotic dependences of  $W_{\text{eff}}(-i\omega)$ <sup>5</sup>:

$$W_{\text{eff}}(-i\omega) = a_0 + a_1(-i\omega)^{1/2} + \dots \quad , \quad \omega \rightarrow 0 \quad , \quad (10)$$

$$W_{\text{eff}}(-i\omega) = b_0 - b_1/(-i\omega) + \dots, \quad \omega \gg W_{\text{max}}, \quad (11)$$

where the expansion coefficients  $a_i(T)$  and  $b_i(T)$  can easily be calculated from Eqs.(6), (7), and (9).

It follows that  $\text{Re } \sigma(\omega, T)$  increases from a dc value,  $\sigma_{\text{dc}}(T) \propto a_0(T)/T$ , to a high frequency value,  $\sigma_{\infty}(T) \propto b_0(T)/T$ . The explicit temperature dependence of  $\sigma_{\text{dc}}$  is given by

$$\sigma_{\text{dc}} \propto \frac{T_m - T}{T^2} \frac{\exp(-\Delta_{\text{max}}/T)}{1 - \exp[-(\Delta_{\text{max}} - \Delta_{\text{min}})(1/T - 1/T_m)]}, \quad (12)$$

and it further turns out that

$$\omega_{\text{c.o.}}(T) \propto T \sigma_{\text{dc}}(T), \quad (13)$$

where  $\omega_{\text{c.o.}}$  denotes the crossover frequency at which  $\text{Re } \sigma(\omega, T)$  starts to increase. The ratio  $\sigma_{\infty}/\sigma_{\text{dc}}$  is large at low  $T$ , and decreases towards one with increasing temperature.

The dielectric constant,  $\epsilon(\omega, T) \propto -\text{Im } \sigma(\omega, T)/\omega$ , is predicted to vary as  $\omega^{-1/2}$  at low frequencies and as  $\omega^{-2}$  at high frequencies. As a function of  $T$ , it exhibits a pronounced peak if  $\omega$  is not too large.

We finally note that the  $(-i\omega)^{1/2}$  term in the low frequency expansion of  $\langle D(-i\omega) \rangle \approx W_{\text{eff}}(-i\omega)$ , Eq.(10), leads to a  $t^{-3/2}$  long time tail in the memory kernel of the associated non-Markovian master equation<sup>11</sup>. If confirmed experimentally, the existence of such long time tails may turn out to become an important aspect of transport in low-dimensional conductors.

#### IV. APPLICATION TO $\text{Qn}(\text{TCNQ})_2$

The results of the preceding section indicate that our model can account, at least qualitatively, for all the features exhibited by  $\sigma(\omega, T)$  in quasi-one-dimensional electronic conductors. To obtain a quantitative description of the complete  $\text{Qn}(\text{TCNQ})_2$  data<sup>3</sup>, however, we have to determine the values of  $\Delta_{\text{min}}$ ,  $\Delta_{\text{max}}$ , and  $T_m$ . From the measured<sup>3</sup> ratios of  $\sigma_{\text{dc}}(300\text{K})/\sigma_{\text{dc}}(80\text{K})$  and  $\sigma_{\infty}(80\text{K})/\sigma_{\text{dc}}(80\text{K})$ , we obtain<sup>5</sup>  $\Delta_{\text{min}} \approx 290\text{K}$  and  $\Delta_{\text{max}} \approx 600\text{K}$ , if  $T_m \approx 320\text{K}$ . The

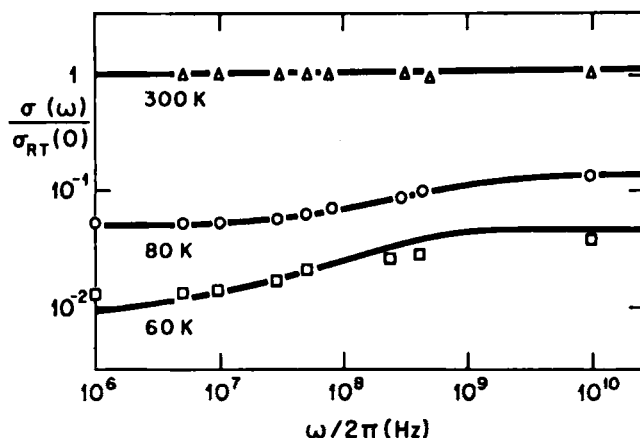


FIGURE 1 Model results for  $\text{Re } \sigma(\omega, T)$  [full curves] vs. frequency for three different temperatures. The model parameters are  $\Delta_{\min} = 290\text{K}$ ,  $\Delta_{\max} = 600\text{K}$ ,  $T_m = 320\text{K}$ , and  $\omega_{\text{at}} = 1.9 \times 10^{11} \text{ sec}^{-1}$ . The experimental data are from Ref.3.

dependence of  $\Delta_{\min}$  and  $\Delta_{\max}$  on  $T_m$ , however, is very weak, and it does not seem possible to determine the precise form of  $\beta(\Delta)$  between its cutoffs from the present experimental data.

With the above parameter values, however, we obtain a reasonably accurate fit to the complete data<sup>3</sup> for  $\sigma(\omega, T)$  in  $\text{Qn}(\text{TCNQ})_2$ . In Fig.1 we present our results for the frequency dependence of  $\text{Re } \sigma(\omega, T)$  at three different temperatures, and in Fig.2 we exhibit the temperature dependence of the dielectric constant,  $\epsilon(\omega, T) \propto -\text{Im } \sigma(\omega, T)/\omega$ , at three different frequencies. In addition, a detailed comparison of the experimental data for  $\sigma_{\text{dc}}(T)$  with Eq.(12), and a verification of the proportionality of  $\omega_{c.o.}(T)$  and  $T\sigma_{\text{dc}}(T)$ , Eq.(13), can be found in Ref.3. Taking into account the simplicity of the model, and the small number of fitting parameters, the overall agreement with the rather complicated behavior of both  $\text{Re } \sigma(\omega, T)$  and  $\text{Im } \sigma(\omega, T)$  seems quite satisfactory.



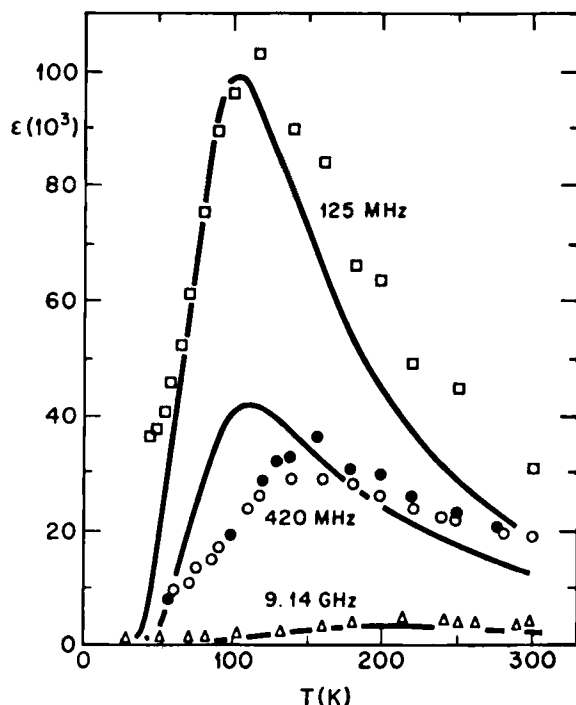


FIGURE 2 Model results for  $\epsilon(\omega, T)$  [full curves] vs. temperature for three different frequencies. Same model parameters as in Fig.1, experimental data from Ref.3.

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## NOTE ADDED IN PROOF

For hopping rate distributions  $\rho(w)$ , for which the moments

$$m_{-k} = \int_0^{\infty} dw \rho(w) w^{-k}, \quad k = 1, 2, \dots,$$

exist, R. Zwanzig (preprint) has recently shown that the first two terms of the low frequency expansion for  $W_{\text{eff}}(-i\omega)$  coincide with the exact low frequency expansion of  $\langle D(-i\omega) \rangle$ :

$$\langle D(-i\omega) \rangle = 1/m_{-1} + \frac{m_{-2} - m_{-1}^2}{2m_{-1}^{5/2}} (-i\omega)^{1/2} + \dots$$

For these type of hopping rate distributions at least, our Effective-Medium approximation thus becomes exact both at high and at low frequencies.